

## Simulation and Numerical Modeling of a Rectangular Patch Antenna Using Finite Difference Time Domain (FDTD) Method

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### Abstract

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This work is to apply the numerical method of Finite Difference Time Domain ( FDTD ) for modeling a rectangular patch antenna via a MATLAB code using UPML formulation for absorption boundary conditions. Reflection coefficient and input impedance of this antenna are determined and compared with simulation results by the HFSS soft.

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**Keywords:** FDTD method, Uniaxial Perfectly Matched Layer formulation (UPML), Reflection coefficient S11 and input impedance  $Z_{in}$  of the antenna

### Introduction

Since its first introduction in 1966, the finite difference time domain (FDTD) method has been widely used as a tool for solving complex electromagnetic problems. For FDTD, Maxwell's equations are discretized in space and time and it is very flexible in modeling complex structures. The first applications of the FDTD were mainly broadcast. But more recently, a number of researchers have applied this method to analyze a wide range of electromagnetic problems: electromagnetic compatibility, imaging subsurface [1], the transmission by radar until the bio-electromagnetic [2] including problems antennas.

The FDTD algorithm can be used to analyze any antenna shape: drawings or microstrip stacked probe or openings coupled.

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The work proposed in this paper is an application of the FDTD method to the study of a rectangular patch antenna. The results of this modeling using Matlab code will be compared to those obtained by HFSS Soft simulation.

## 1. Formulation of FDTD Method

The FDTD method is based on the numerical solution of Maxwell's equations. It consists in approximating the punctually derived spatial and temporal that appear in these equations by centered finite differences [3].

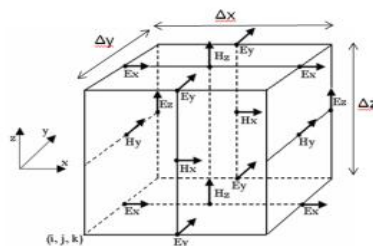
For UPML formulation, the study space is considered as anisotropic medium [4] and the Maxwell equations are expressed as follows :

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} \quad (1)$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad (2)$$

Where  $\vec{E}$  and  $\vec{H}$  are the electric and magnetic fields.  $\bar{\epsilon}$  is the diagonal tensor permittivity.

Spatial discretization is achieved by subdividing the computation space in cells called Yee cells (Fig.1). Each cell is marked by indices (i, j, k) and whose space steps  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  are depending on the smaller wavelength in the analysis frequency band of the structure under test.



**Figure 1 : Implementation of the Components of the Electric and Magnetic Fields in a Cell Yee**

In a cell, each magnetic node is surrounded by four electrical nodes and each electric node is surrounded by four magnetic nodes. So centered derivative is used for all spatial derivatives present in the Maxwell equations.

For uniform time discretization, with a sampling step  $\Delta t$ , and an index  $n$  that corresponds to the real time  $n.\Delta t$ . In other side components of the electric field and electric flux are calculated for multiple peers of  $\Delta t/2$ , whereas the components of the magnetic field and magnetic flux are calculated for odd multiples of  $\Delta t/2$ .

Thus, the ratings will be used as in the following two examples :

- For electric field component  $E_z \Big|_{i,j,k+0.5}^n \rightarrow E_z(i\Delta x, j\Delta y, k\Delta z + 0.5\Delta z; n\Delta t)$
- For magnetic field component  $H_y \Big|_{i+0.5,j,k+0.5}^{n+0.5} \rightarrow H_y(i\Delta x + 0.5\Delta x, j\Delta y, k\Delta z + 0.5\Delta z; n\Delta t + 0.5\Delta t)$

## 2. Rectangular Patch Antenna Study

we proposed to apply the FDTD method to modeling a rectangular patch antenna whose dimensions are shown in Figure 2 [5]. So we are interested to resolving 12 equations concerning field components variation that are evaluated in the following order  $D_x, D_y, D_z, E_x, E_y, E_z, B_x, B_y, B_z, H_x, H_y, H_z$ .

The computational space is composed of two parts, one for the antenna and the second for the conditions UPML which is surrounded by surface type PEC (Perfect Conductor Electric), where the electric field must be zero which allows the updating of the fields in the calculation algorithm.

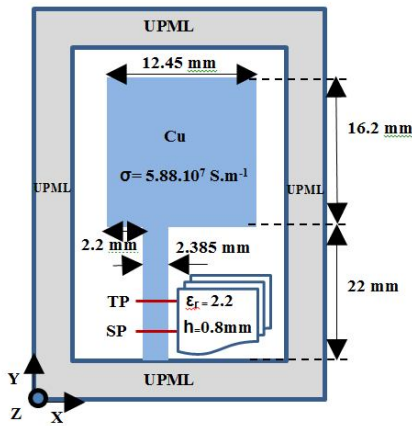


Figure 2. Structure of Rectangular Patch

The stability condition for the algorithm of the FDTD method is given by the inequality (12), [4]

$$\Delta \leq \frac{\lambda_{min}}{10} \text{ and } \Delta t \leq \frac{\Delta}{c_0 \sqrt{n}} \quad (12)$$

Or  $\lambda_{min}$  is minimum wave length corresponding to the maximum frequency supported by the structure being studied and the  $C_0$  is the velocity of light in vacuum. Thus, for three-dimensional ( $n = 3$ ), we can take  $\Delta t = \frac{\Delta}{2c_0}$ .

For the excitation source in the SP plan (Source Plan), we can choose a Gaussian pulse whose function is given by the expression (13). It will be added to all components  $E_z$  in the FDTD algorithm. With  $t_0 = 120 \cdot \Delta t$  and  $t_w = 30 \cdot \Delta t$  in order to generate a wave in TEM mode.

$$E_z(t) = \exp \left( - \left[ \frac{t-t_0}{t_w} \right]^2 \right) \quad (13)$$

The maximum supported by the studied structure frequency is about 50GHz, spatial and temporal steps discretization adopted for this case will be  $\Delta = 0.256 \text{ mm}$  et  $\Delta t = 0.441 \text{ ps}$ .

Where the space problem has the size of 70x16x150 cells. UPML conditions occupies 19 cells for each of six face. Therefore the size of the computing space is **106x186x52 cells**.

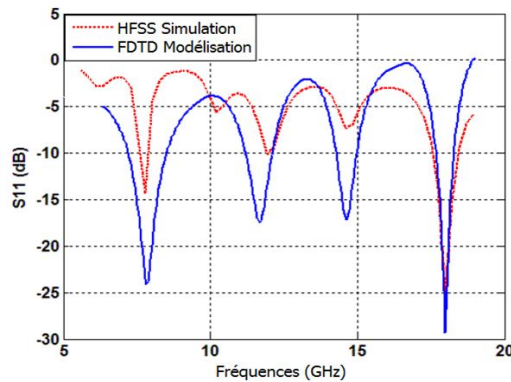
The modeling algorithm for this antenna uses 30 coefficients existing in 12 equations in order to evaluate electric and magnetic components fields. This requires in total **42 three dimensional matrices**, where each matrix is the **size 106x186 x 52**.

The result of this modeling is the reflection coefficient  $S_{11}$ . At Terminal plane (TP), shown in Figure 2 and which is located at a distance of  $10 \cdot \Delta$  for source plane (SP) level, we estimate the value of the coefficient  $S_{11}$ , that the expression in (14), by revaluing incident and reflected wave given by the components  $E_z$  or  $H_x$ .

$$S_{11}(f) = \frac{E_{ref}(f)}{E_{inc}(f)} = \frac{H_{ref}(f)}{H_{inc}(f)} = \frac{FT\{H_{ref}(t)\}}{FT\{H_{inc}(t)\}} \quad (14)$$

$$S_{11}(dB) = 10\log (|S_{11}|) \quad (15)$$

To obtain  $S_{11}(f)$ , we used Fourier transformation which is given by the MATLAB (Fast Fourier Transform) function (FFT). Then we evaluated the variation of the reflection coefficient  $S_{11}(dB)$ , whose expression in (15) as a function of the frequency. The graph is shown in Figure 3. On the same Figure also are shown the results of the simulation by the HFSS Soft.



**Figure 3. Variation Coefficient  $S_{11}$  Depending on the Frequency for Rectangular Patch Antenna**

It is noted that the two curves have the same shape and magnitude is approached. The slight difference may be explained by the distortion of source which must be carefully chosen.

By comparing the resonance frequencies from the cavity model, we find that the resonance frequency of the antenna is usable in  $TM_{020}$  mode is given by:

$$:(f_r)_{020} = \frac{c_0}{2w\sqrt{\epsilon_r}} = 8.1 \text{ GHz.}$$

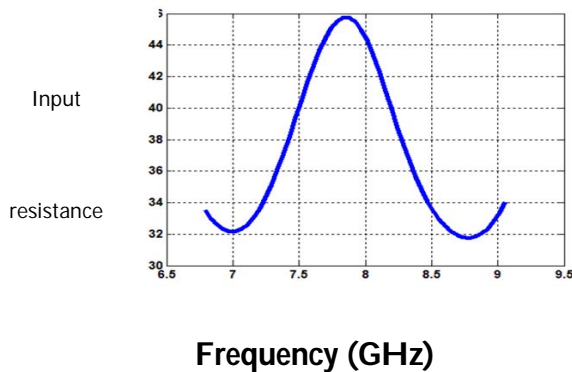
In Figure 3, we see that the exploitable resonance frequency is about 7.8 GHz. The slight difference from cavity model may be explained by the fact that the cavity model is an approximate model and FDTD modeling and HFSS simulation gives more accurate results.

On the other hand, the input impedance of the patch antenna at terminal plan (TP) is related to the reflection coefficient  $S_{11}$  by expression (18)

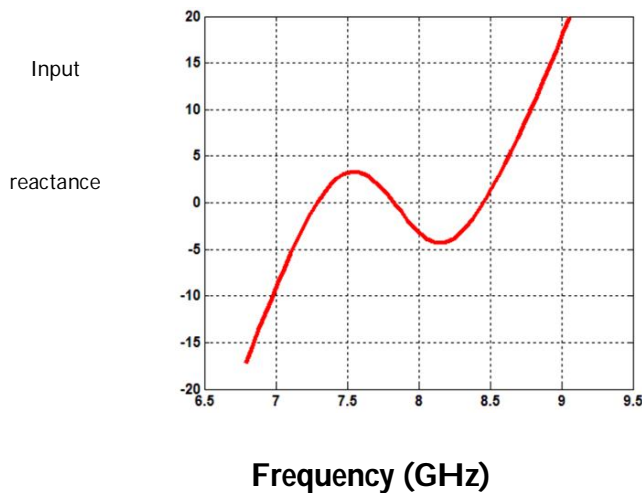
$$Z_{in}^{TP} = Z_0 \left( \frac{1+S_{11}}{1-S_{11}} \right) \quad (18)$$

With  $Z_0 = 50\Omega$  is the characteristic impedance of the feed line.

The variation of the resistance and the reactance according to the frequency are shown in Figures 4 and 5.



**Figure 4. Input Resistance Depending on the Frequency for the Line Fed Rectangular Patch Antenna**



**Figure 5. Input Reactance Depending on the Frequency for the Line Fed Rectangular Patch Antenna**

The Figure4 shows that at the resonance frequency of 7.8Ghz, Input resistance is approximately 46Ω. This value is close to the characteristic impedance of the feed line which is 50Ω

From Figure 5, it is found that the reactance at the resonant frequency is zero and that is ideal for the resonance. Since the input impedance of the patch antenna in the terminal plane (TP) is known, we can estimate antenna input impedance  $Z_{in}$  defined by (19).

$$Z_{in} = Z_0 \left[ \frac{Z_{in}^{TP} - jZ_0 \tan \beta l}{Z_0 - jZ_{in}^{TP} \tan \beta l} \right] \quad (19)$$

$\beta$  (rad/m) is the phase constant of the line fed,  $v_p$  phase velocity and  $\epsilon_{reff}$  effective dielectric constant can be calculated from the expressions (20), (21) and (22).

$$\beta = \frac{2\pi f}{v_p} \quad (20)$$

$$v_p = \frac{v_0}{\sqrt{\epsilon_{reff}}} \quad (21)$$

$$\epsilon_{reff} = \left( \frac{\epsilon_r + 1}{2} \right) + \left( \frac{\epsilon_r - 1}{2} \right) \frac{1}{\sqrt{1 + 12h/w_f}} \quad (22)$$

### 3. Conclusion

In this article, we applied the FDTD method with the UPML formulation for modeling a rectangular patch antenna excited by a Gaussian temporal source. Results are in good agreement with those of the HESS simulation. This allows us to validate the FDTD method for the study of rectangular patch antennas. This model can be extended to the study of other antennas of different form or for analysis array antennas.

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