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Efficient Detection of Multi-Narrowband Using the Warped Discrete Fourier Transform

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Abstract

This paper presents a multi-narrowband signal processing paradigm that is based on the use of the warped discrete Fourier transform (WDFT). The WDFT evaluates a discrete-time signal in the context of a non-uniform frequency spectrum, a process called warping. Compared to a conventional DFT or FFT, which produces a spectrum having uniform frequency resolution across the entire baseband, the WDFT's frequency resolution is both non-uniform and programmable. This feature is exploited for use in analyzing multi-narrowband signals which are problematic to the DFT/FFT. The paper focuses on optimizing frequency discrimination by determining the best warping strategy and control by using the intelligent search algorithms and criteria of optimization, or cost functional. The system developed and tested focuses on maximizing the WDFT frequency resolution over those frequencies that exhibit a localized concentration of spectral energy and, implicitly, diminishing the importance of other frequency ranges. The paper demonstrates that by externally controlling the frequency resolution of the WDFT in an intelligent manner, multi-narrowband signals can be more readily detected and classified. Furthermore, the WDFT can be built upon an FFT enabled framework, insuring high efficiency and bandwidths.

Keywords: Signal analysis, Frequency discrimination, Nonuniform, Warped discrete Fourier transform (WDFT), Spectral leakage

1. Introduction

Multi-narrowband signal analysis including detection and discrimination is a continuing signal processing problem. The applications include dual-tone multi-frequency (DTMF) systems, Doppler radar, electronic countermeasures, wireless communications, OFDM-based radar exciters, to name but a few. Traditional multi-narrowband signal analysis systems are based on a filterbank architecture that uses an array of product modulators to heterodyne signals down to DC, and then processes the down-converted array of signals with a bank of lowpass filters. The output of the filterbank is then processed using a suite of energy detection operations to detect the presence of tones and multiple narrowband tones (Vassilevsk, 2007). The capability of such a system to isolate and detect multiple narrowband signals is predicated on the choice of initial modulating frequencies and post-processing algorithms. Other approaches to the problem are based on multiple signal classification (MUSIC) algorithms, least mean-square (LMS) estimators, and DFT derivatives such as Goertzel algorithm (Evans, 1996). The approach taken in this paper is to explore the use of another DFT derivative called the *warped discrete Fourier transform* or *WDFT*(Makur et al., 2001; Franz et al., 2003).

2. FFT- The Enabling Technology

The discrete Fourier transform(DFT) is indisputably an important signal analysis tool, finding applications in virtually all engineering and scientific endeavors. Generally, the preferred implementation of the DFT is the venerable Cooley-Tukey fast Fourier transform (FFT) algorithm.

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An N-point DFT $X[k]$, $0 \leq k \leq N$-1, of a length-N time-series $x[n]$, $0 \leq n \leq N$-1, is defined by:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \qquad 0 \leq k \leq N-1. \qquad (1)$$

For spectral analysis applications, the DFT provides a uniform frequency resolution $\Delta = 2\pi/N$ over the normalized baseband $\omega \in [-\pi, \pi]$. That is, the DFT's frequency resolution Δ is uniform across the entire baseband. This fact historically has limited the role of the DFT in performing acoustic and modal (vibration) analysis, applications that prefer to interpret a signal spectrum using logarithmic (octave) frequency dispersions. Another application area in which a fixed frequency resolution is a limiting factor is multi-narrowband signal detection and classification. It is generally assumed that if two tones are separated by 1.6Δ (1.6 harmonics) or less, then a uniformly windowed DFT/FFT cannot determine if one tone or multiple tones are present at a harmonic frequency due to spectral leakage, which obscures the spectral separation between adjacent DFT harmonics (Mitra et al., 1993). This problem is exacerbated when data widows are employed (*e.g.*, Hamming window). This condition is illustrated in Fig. 1. In Fig. 1(a), two tones separated by one harmonic (i.e., Δ) are transformed. The output spectrum is seen to consist of a single peak, losing the identification of each individual input tone because their main lobes get closer and eventually overlap. In the other case reported by Fig. 1(b), the two tones being transformed are separated in frequency by two harmonics (i.e., 2 Δ). The presence of two distinct tones is now self-evident.

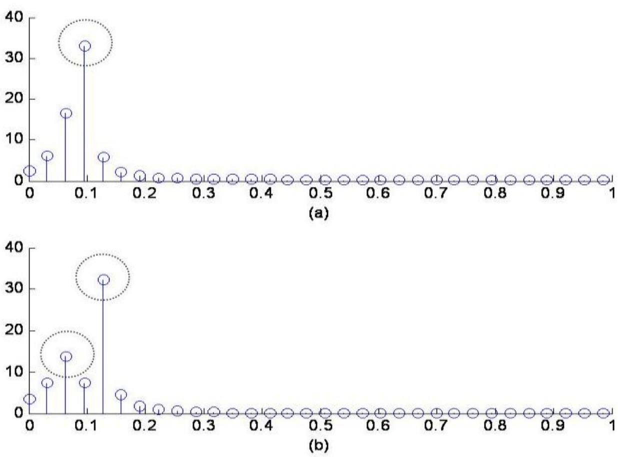


Figure 1: Magnitude spectrum for two tones using a 64-point DFT. (a) Two tone separated by one harmonic. (b) Two tones separated by two harmonics.

3. The Warped Discrete Fourier Transform (WDFT)

The WDFT is a derivative of the familiar DFT filterbank (Taylor et al., 1999). It differentiates itself from the standard DFT filterbank in that it contains an additional pre-processed stage. The WDFT can be developed in the context of multi-rate and polyphase signal processing theory (Mitra, 2001). A polyphase multi-rate filter architecture, shown in Fig. 2, was used to implement a WDFT (Galijasevic et al., 2002). In Fig.2, the filter function $A(z)$ is as a pre-processing all-pass filter. For the case where $A(z)$=1, the design degenerates to a traditional uniform DFT filterbank (Taylor et al., 1999;Mitra, 2001). For the case where $A(z)$=1 and the polyphase terms $P_i(z)$=1, the architecture shown in Fig. 2 becomes an N-point DFT.

Formally the N-point WDFT, reported in Fig. 2 for $P_i(z)$=1, is defined in terms of a DFT and pre-processing the all-pass filtered data, filtered by $A(z)$, where:

$$A(z) = \frac{-a + z^{-1}}{1 - az^{-1}} \qquad (2)$$

where "a"is real and is called the *warping control parameter*. For stability reasons, "a" ranges between -1 and 1.

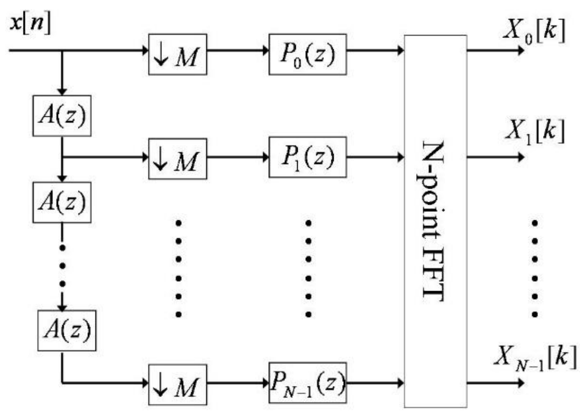
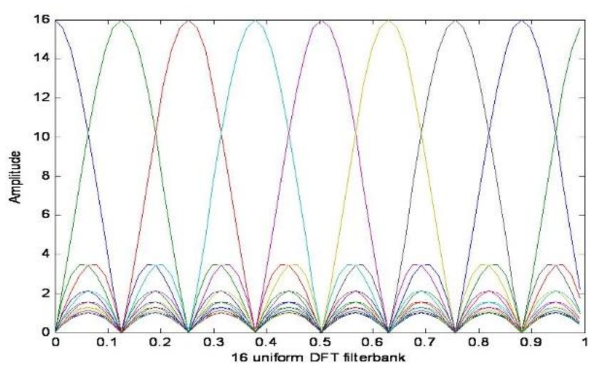


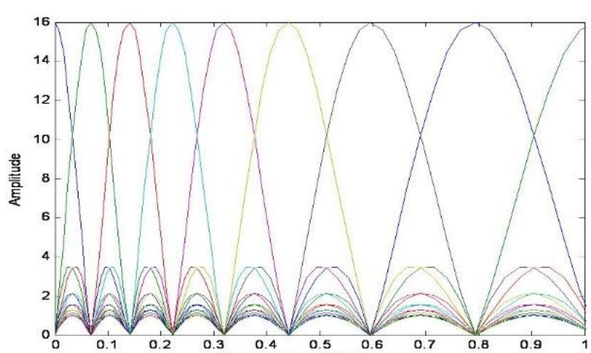
Figure 2. Polyphase multi-rate FFT Filterbank.

To motivate the behavior of a WDFT, consider the experiment reported in Fig.3.The uniform resolution DFT case ($P_i(z)$=1 and *a*=0) is compared to the non-uniform resolution case ($P_i(z)$=1 and *a* = ±0.3). The ability of locally control the frequency resolution of the WDFT is clearly demonstrated. In addition, if a lowpass subband shaping filter polyphase filter ($P(z)$=

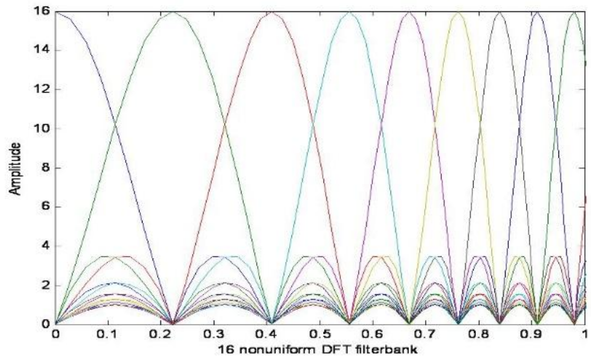
$\Sigma z^{i} P_i(z)$) is employed (*e.g.*, approximate ideal lowpass FIR), then additional control can be exercised over the shape and frequency selectivity of the WDFT spectrum.



(a)



(b)



(c)

Figure 3. WDFT experiments: $\overline{\omega}$ corresponds to normalize baseband in [0,π]. 16-channel (a) uniform FFT spectrum, (b) nonuniform FFT spectrum for *a* = -0.3, (c) nonuniform FFT spectrum for *a* = 0.3.

Concentrating on the WDFT case where $P(z)=1$, it may be recalled that the standard z-transform of an N-point input time series $x[n]$, namely

$$X(z) = \sum_{n=0}^{N-1} x[n]z^{-n}. \quad (3)$$

The warping filter converts the input into:

$$\bar{X}(z) = \sum_{n=0}^{N-1} x[n]A(z)^{n}. \quad (4)$$

Recall that the conventional DFT of $x[n]$, namely $X[k]$, produces a spectrum given by:

$$X[k] = X(z)|_{z=e^{j2\pi k/N}}, \qquad 0 \le k \le N-1. \quad (5)$$

where $X[k]$ is evaluated at $z=e^{j2\pi k/N}$, a point the uniformly resolved locations on the periphery of the unit circle in the z-domain. The WDFT coefficients, $\bar{X}[k]$, are similarly obtained by uniformly sampling $\bar{X}(z)$ at points on the unit circle in the z-domain, namely:

$$\bar{X}[k] = \bar{X}(z)|_{z=e^{j2\pi k/N}}, \qquad 0 \le k \le N-1. \quad (6)$$

The conventional uniform frequency resolution DFT, defined by $z = e^{j\omega}$, has harmonic frequencies located at frequencies $\omega = 2\pi k / N$, $k \in [0, \ldots, N-1]$. The center frequencies of an N-point WDFT spectrum are located at the warped frequencies $\overline{\omega}$ where $z = e^{j\overline{\omega}}$, which are associated to ω through the non-linear frequency warping relationship

$$\tan\left(\frac{\overline{\omega}}{2}\right) = \left(\frac{1-a}{1+a}\right)\tan\left(\frac{\omega}{2}\right). \quad (7)$$

Equation (7) establishes a non-linear frequency warping relationship that is controlled by the real parameter "*a*". A positive "*a*" provides higher frequency resolution on the high frequency region and a negative value of "*a*" increases frequency resolution in the low frequency region (see Fig. 3).

The effect of the warping relationship is demonstrated in Fig. 4 which compares a DFT (a = 0) to WDFTs for a = -0.071, a = -0.23 and a = -0.4 for the case where two tones are present separated by a single DFT harmonic. It is easily seen that by intelligently choosing the control parameter "*a*" the locally imposed frequency resolution can be expanded or contracted. To enhance the system's frequency discrimination, the frequency resolution should be maximized in the local region containing the input signals. As such, an intelligent agent will need to assign the best warping parameter "*a*" strategy, one that concentrates the highest frequency resolution in the spectral region occupied by the multi-narrowband process.

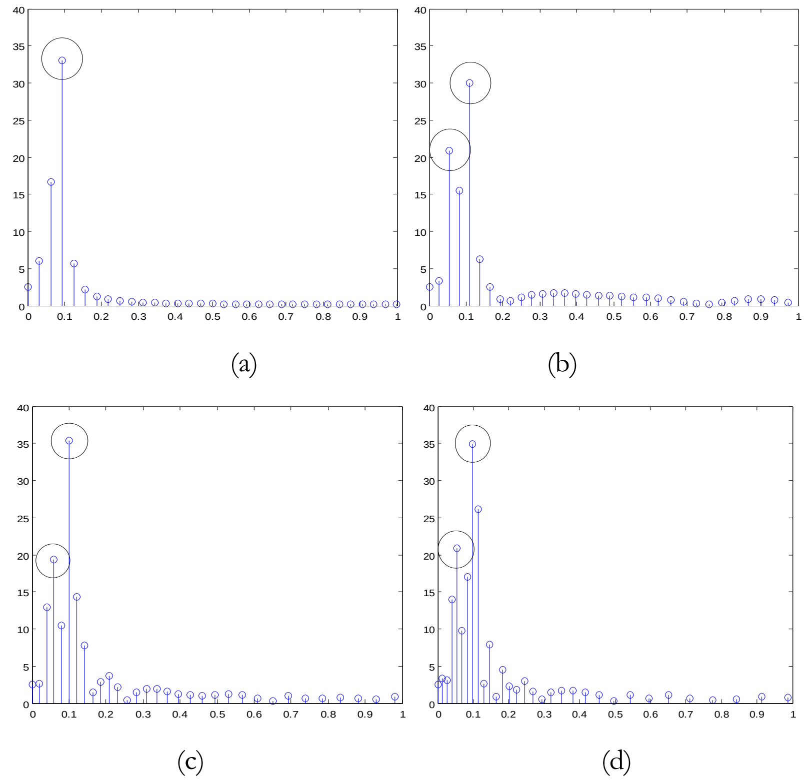


Figure 4. Magnitude spectrum for two tones with 64-point WDFT with (a) $a = 0$, (b) $a = -0.071$, (c) $a = -0.23$, and (d) $a = -0.40$.

The next section describes the outcome of a preliminary study that compares two criteria and two search algorithms and develops an "intelligent" frequency resolution discrimination policy that can be used to improve multi-narrowband detection.

4. Optimization of Frequency Resolution

To optimize the choice of the warping parameter "*a*", $|a|<1$, an intelligent search algorithm or agent is needed. An initial search strategy is being evaluated and enabled using optimal single-variable search techniques, a Fibonacci search[2](Kwon et al., 2008)and a modified Golden Section search[3](Kwon et al., 2012). The search process is expected to iterate over a range of values of "*a*" that places a high local frequency resolution in the region occupied by multi-narrowband activity. To find the best warping parameter "*a*", two criteria of optimization and cost functionals have been singled out for focused attention. The search methods iteratively restrict and shift the search range so as to optimize spectral resolution within a convergent range. The direction of the search is decided by the value of the cost functional at two points in the range.
Two criteria studied to date are developed below.

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[2]The Fibonacci search technique is a method of searching a sorted array using a divide and conquer algorithm that narrows down possible locations with the aid of Fibonacci numbers.
[3]The Golden Section search is a technique for finding the extremum (minimum or maximum) of a unimodal function by successively narrowing the range of values inside which the extremum is known to exist.

A. Criterion #1

$$\Phi_1(\bar{\omega}) = \max_k[\bar{X}[k]] - \sum[\bar{X}[k]|_{\bar{\omega}_b}] \quad (8)$$

where $\bar{\omega}_b$ is a frequency within the search interval and $\Phi_1(\bar{\omega})$ is designed to reward a local concentration of spectral energy and penalize more sparsely populated section of the spectrum. The optimal operating point corresponds to a warping parameter "a" that maximizes the local spectral resolution in a region of signal activity.

B. Criterion #2:

$$\Phi_2(\bar{\omega}) = \sum[\sigma - \bar{X}[k]] \quad (9)$$

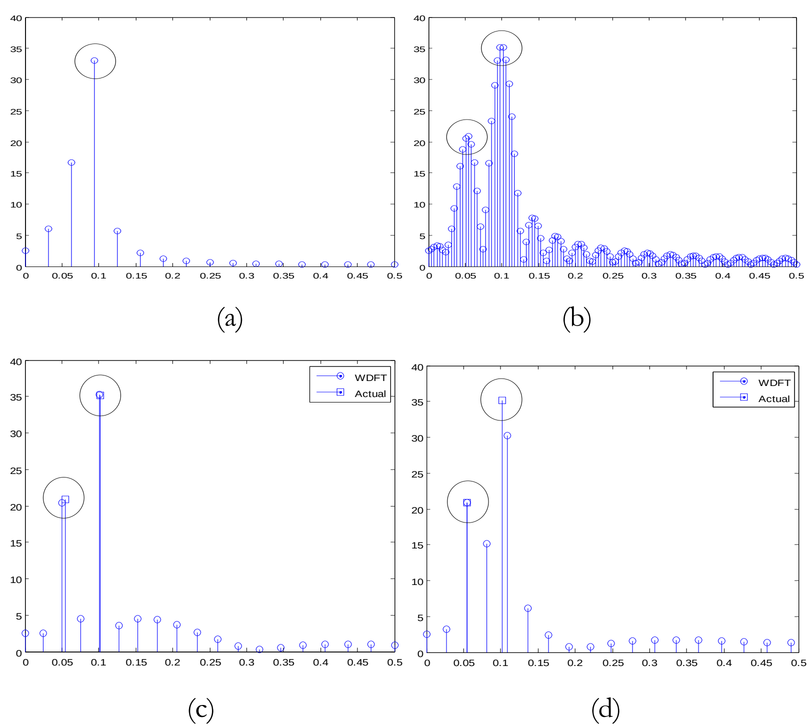
where σ is the threshold used to suppress leakage and $\Phi_2(\bar{\omega})$ is designed to reward the local concentration of spectral energy.

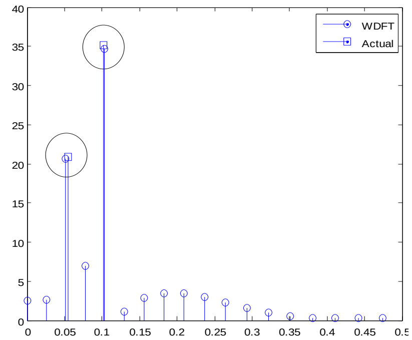
Table 1 Comparison of the warping parameter

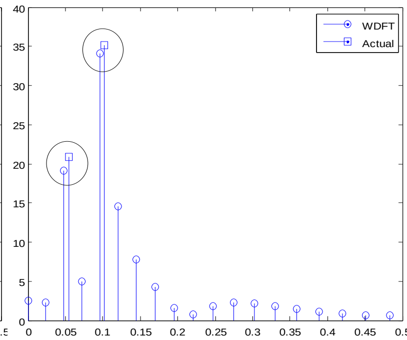
Criterion	Criterion #1		Criterion #2	
Search Method	Fibonacci search	M.Golden section search	Fibonacci search	M.Golden section search
Warping parameter	*a* = -0.1087	*a* = -0.0721	*a* = -0.0996	*a* = -0.1381
Elapsed time (sec)	t = 0.749252 s	t = 0.888743 s	t = 0.736151 s	t = 0.151035 s

5. Results and Comparison

The paper reports on a multi-narrowband signal discrimination study conducted using two search methods, namely a Fibonacci search and a modified Golden Section search algorithm. Both are iterative methods that restrict and shift the searching range so as to determine an optimal operating point within a frequency range. Studies based on these criteria involved presenting to the WDFT frequency discriminator of two sinusoidal tones located at 0.157 and 0.314 rad/s. The search method was charged to find the "best" warping parameter. The comparison of results is shown in Table 1 and the evidence of this activity can be seen in Fig. 5. To compare the temporal efficiency of each case, Table 1 also shows elapsed time needed to execute a search using MATLAB. The two tones, separated by one harmonic, were unresolved with 64-point DFT but resolved with 512-point DFT at the expense of increased complexity (see Fig. 5 (a) and (b), respectively). In Fig. 5 (c)-(e), however, the two tones are seen to be present using 64-point WDFT. To calibrate the WDFT spectra, the locations of the actual two tones are also shown. Comparing the outcomes, a Fibonacci search







(e) (f)

Figure 5. (a) Magnitude spectrum for two tones with (a) 64-point DFT (*a* = 0) and (b) 512-point DFT (*a* = 0) and magnitude spectrum for two tone detection with 64-point WDFT with (c) *a* = -0.1087, (d) *a* = -0.0721, (e) *a* = -0.0996 and(f) *a* = -0.1381.

was found to be the fastest and most effective in finding the "best" warping parameter using either search criteria. *Criterion #1* resulted in frequency resolution with a bigger variation according to search methods, while *Criterion #2* facilitated the optimization of the local resolution and identified the two tones closer to the actual locations of the tones.

6. Conclusion

This paper aims at exploring spectral analysis using the warped discrete Fourier transform (WDFT) compared to a conventional discrete Fourier transform (DFT). It focuses on detecting multiple narrowband signals which are not able to be isolated with uniform frequency resolution and optimizing the local frequency resolution by finding the best warping control strategy. And the system developed and tested also focuses on maximizing the WDFT frequency resolution over those frequencies that exhibit a localized concentration of spectral energy and, implicitly, diminishing the importance of other frequency ranges. This paper demonstrates that multi-narrowband signals are able to be more readily detected and discriminated by externally controlling the frequency resolution of the WDFT in intelligent manners using optimal single-variable search techniques, a Fibonacci search and a modified Golden Section search.
Finally, this paper shows the best frequency resolution reducing spectral leakage which obscures the spectral separation between of adjacent DFT harmonics due to the finite frequency resolution of the DFT without increasing the DFT length for multi-narrowband detection using the WDFT. In fact, an increase in the DFT length improves the sampling accuracy by reducing the spectral separation of adjacent DFT samples, while it brings up the higher computational complexity and cost penalty. In order to minimize and suppress spectral leakage the WDFT is exploited to control the spectral separation through the warping parameter.

Moreover, the usage of the WDFT presents obtaining higher and optimized local frequency resolution through finding the best warping control strategy. In general, a uniformly windowed DFT/FFT cannot determine if one tone or multiple tones are present locally about a harmonic frequency for two tones separated by 1.6 Δ (1.6 harmonics) or less. It shows that, however, the WDFT can discriminate between two signals separated by as little as 1.3 harmonics. Overall, the new spectral analysis technology using the WDFT results in a higher local resolution, less computational complexity, more capability, and lower cost.

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